Intermittent turbulence in a Rayleigh-Bénard problem studied using Schwarz domain decomposition

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Introduction and motivation

In fluid mechanics, even simple problems and models may generate complex phenomena. And there is nothing as complex, yet beautiful, as turbulence. Our goal is to study the transition to turbulence for a Rayleigh-Bénard convection problem in a rectangular domain. But this transition is not instantaneous; rather, laminar and turbulent structures alternate in what is called intermittent turbulence. Intermittency can be obseved for Rayleigh numbers (*Ra*) between $2{,}5 \cdot 10^5$ and $5 \cdot 10^5$.

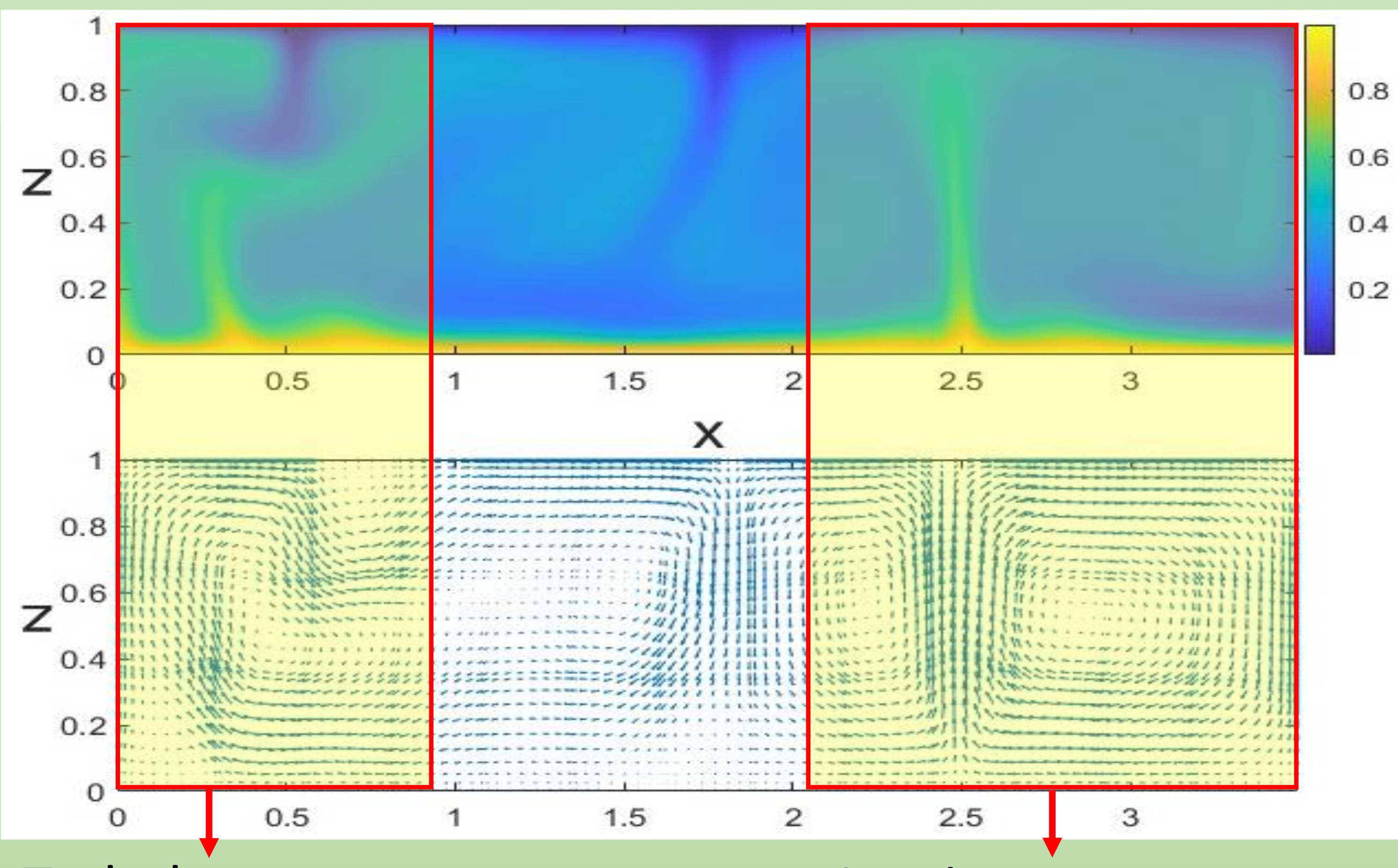
Methodology

The problem is first considered in its dimensionless form, as shown on the right. Then, a time discretitation is applied. Finally, each temporal iteration is solved using a Legendre collocation method.
However, as Legendre collocation is a spectral method, it is ill-conditioned. So it would not be able to reach high *Ra* if it was not for the use of a Schwarz domain decomposition method.

Solutions

Numerical solutions show that turbulent intermittency occurs both in time and space. Temporarily, it can be seen how a turbulent regime becomes completely laminar, and after some period of time, it breaks into turbulence again (Figures on the right).
Additionally, both laminar and turbulent structures can be found at the same time in different sections of the fluid (Figure at the bottom).
As *Ra* increases, intermittency disappears, giving way to a completely turbulent regime.

Spatial intermittency



Turbulent structures:
- Short-lived eddies
- Chaotic thermal plumes

Laminar structures:
- Structured rolls
- Fixed thermal columns

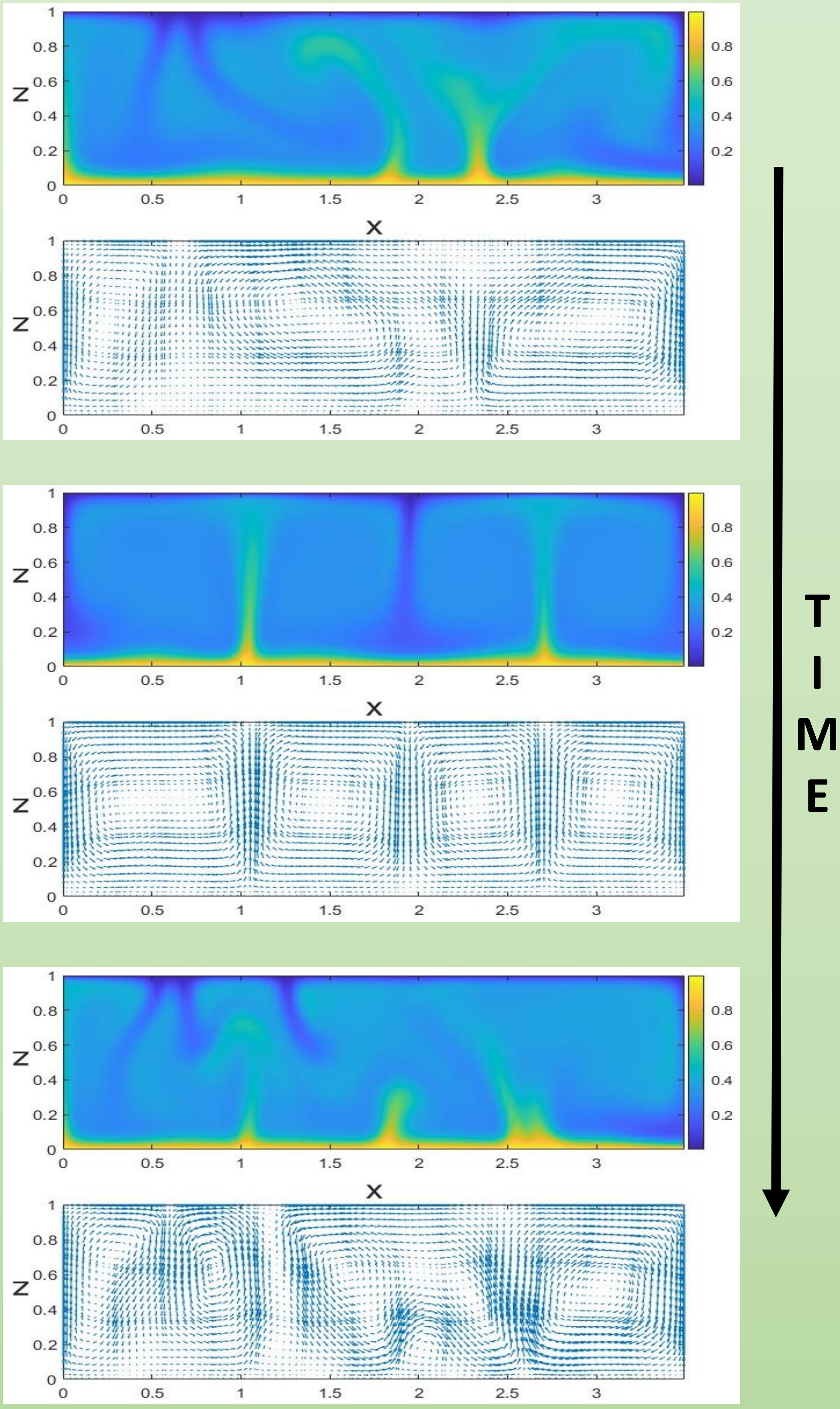
Equations

Continuity equation	$\nabla \cdot \boldsymbol{u} = 0$
Motion equations	$\Delta \boldsymbol{u} - \nabla P + Ra \cdot \theta\, e_z = 0$
Heat equation	$\partial_t \theta + \nabla \theta - \Delta \theta = 0$
Boussinesq approximation	$\boldsymbol{u} \cdot \rho = \rho_1\, (1 - \alpha\, T - T_1\,)$

Boundary conditions

$\boldsymbol{u} - 1 = \theta - 1 = 0$	$in\ z = 0$
$\partial_z\, u_x = u_z = \theta = 0$	$in\ z = 1$
$\partial_x \theta = u_x = \partial_x u_z = 0$	$in\ x = 0,\, L$

Temporal intermittency



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